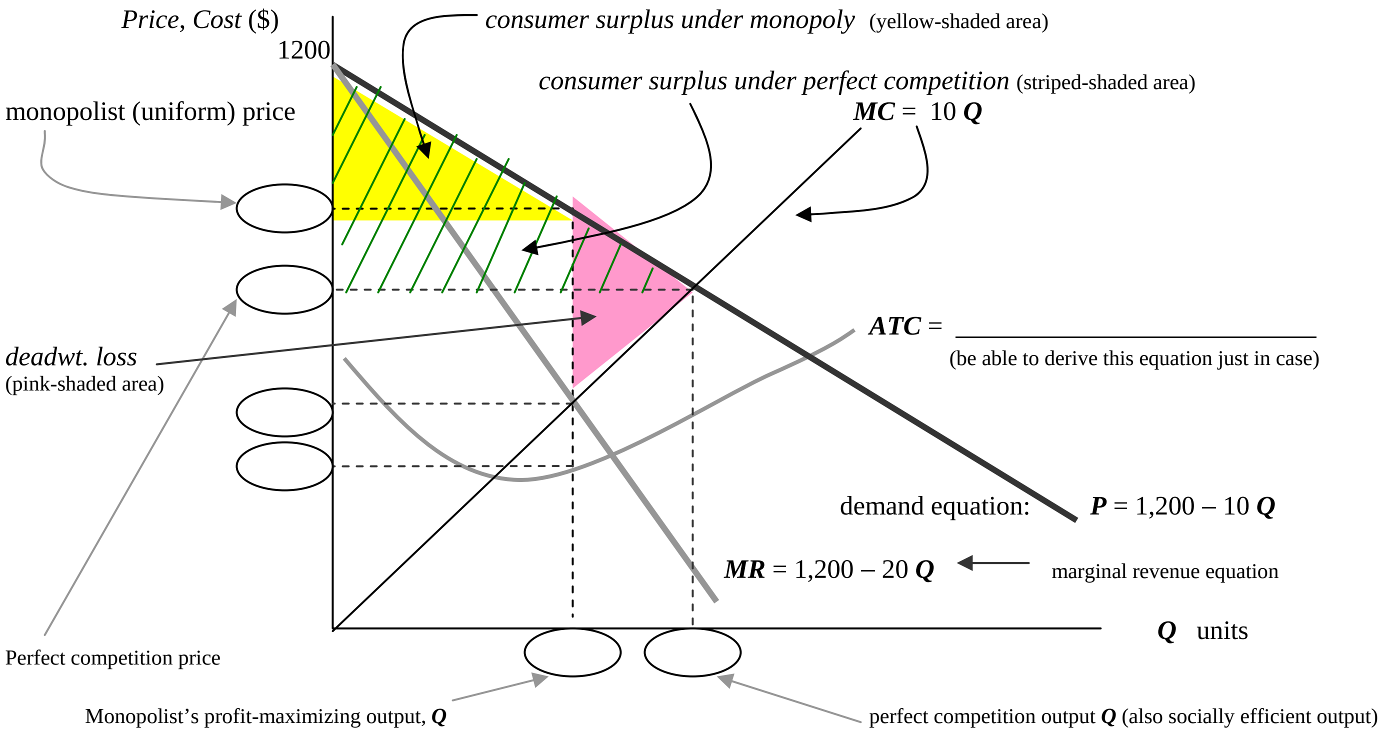
Microeconomics

Ch. 15 Monopoly and Perfect Competition Compared, Price Discrimination *(Practice Problem Solutions)*

Consider a monopolist below in Fig. 1 with the following cost curves (*not* drawn to scale):

Total Cost: $TC = 1{,}000 + 5\,Q^2$

Fig. 1. The Cost Curves of a Monopolist



Part 1

1. What is the profit-maximizing level of output Q for the monopolist? Q = ________ units

(*Hint*: Set $MC = MR$ or $MR = MC$ and solve for Q.) (Answer: 40 units)

Write down your solution in this space here. Prove to yourself that the answer shown is correct.
Then check your solutions with those found on p. 3 under "Detailed Solutions."

2. At the profit-maximizing output Q, what price P is the monopolist charging for its product? Monopolist price, P = $ _____

(It's the price *on* the demand curve corresponding to the monopolist's profit-maximizing level of output Q.) ($800)

(*Hint*: Plug Q from the previous problem into the demand equation and solve for P.)

3. Given its profit-maximizing Q, how much is the monopolist's profit, π,? Monopolist profit, π = $ _________

($23,000)

(*Hint*: First solve for total cost TC given Q from the previous problems, which should be $9,000, then subtract from total revenue (P)(Q).)

Alternatively, without solving for price P but given Q, solve for profit using $\pi = TR - TC = (P)(Q) - [1{,}000 + 5(Q)(Q)] = (Q)(1{,}200 - 10\,Q) - [1{,}000 + 5(Q)(Q)]$.

4. If the monopoly firm were run *not* by a profit-maximizing owner but by a benevolent social planner, one who would produce at a level of ***Q*** equivalent to that of a perfectly competitive output, what would this output be (called *socially efficient quantity*, ***Q***)?
(*Hint*: Set ***MC*** = ***P*** and solve for ***Q*** .)

Socially efficient ***Q*** = __________
(perfectly competitive output) (60 units)

5. How much would this benevolent social planner charge as its price if it is producing the socially efficient output ***Q***? ***P*** = $_____
(*Hint*: Plug ***Q*** from the previous problem into the demand equation and solve for ***P***.) ($600)

6. Following from the two previous questions above, what would be the profit, π, under this situation? π = $ _________
($17,000)

(*Hint*: First solve for total cost ***TC*** given ***Q*** from the previous problems, which should be $19,000, then subtract from total revenue (***P***)(***Q***).)

7. Compare the three outcomes in terms of output ***Q***, price ***P*** and profit π (one under a monopoly (uniform pricing) condition, another under a perfectly competitive condition, and still another under a monopoly (*perfect price* discrimination condition).

Table 1	Part 1 **Monopoly** (uniform pricing)	Part 1 **Perfect Competition** (socially efficient) (or benevolent social planner)	Part 2 **Monopoly** (perfect price discrimination)
Output, ***Q*** (units)	________ (40 units)	________ (60 units)	________ (60 units)
Price charged, ***P*** ($)	________ ($800)	________ $600)	________ (reservation price)
Profit, π ($)	________ ($23,000)	________ ($17,000)	________ ($35,000)
Consumer Surplus, ($)	________ ($8,000)	________ ($18,000)	________ (0)
Producer Surplus, ($)	________ ($24,000)	________ ($18,000)	________ ($36,000)
Total Surplus, ($)	________ ($32,000)	________ ($36,000)	________ ($36,000)
Deadweight Loss, ($)	________ ($4,000)	________ (0)	________ (0)

For detailed calculations of the numbers on this column, please see p. 5.

Detailed Solutions (Part 1)

1. Solving for the profit-maximizing level of output $\boldsymbol{Q}$ for the monopolist.

$$MC = MR$$
$$10\,\boldsymbol{Q} = 1{,}200 - 20\boldsymbol{Q}$$
$$10\,\boldsymbol{Q} + 20\,\boldsymbol{Q} = 1{,}200 \qquad \text{after adding } 20\boldsymbol{Q} \text{ to both sides of the equal sign}$$
$$30\,\boldsymbol{Q} = 1{,}200$$
$$\boldsymbol{Q} = 1{,}200/30$$
$$\boldsymbol{Q} = 40 \text{ units}$$

2. Solving for the (*uniform*) price $\boldsymbol{P}$ charged by the monopolist while producing at its profit-maximizing level of output $\boldsymbol{Q}$.

$$\boldsymbol{P} = 1{,}200 - 10\boldsymbol{Q}$$
$$= 1{,}200 - 10(40 \text{ units}) \qquad \text{after plugging in } \boldsymbol{Q} = 40 \text{ units}$$
$$= 1{,}200 - 400$$

Note: If you don't know how your calculator works, do your computations piece by piece

$$\boldsymbol{P} = \$800$$

3. Solving for the monopolist's profit, π.

$$\pi = \text{TR} - \text{TC}$$
$$= (\boldsymbol{P})(\boldsymbol{Q}) - [1{,}000 + 5(\boldsymbol{Q})(\boldsymbol{Q})]$$
$$= (\$800)(40 \text{ units}) - [1{,}000 + 5(40 \text{ units})(40 \text{ units})] \qquad \text{after plugging in } \boldsymbol{P} = \$800 \text{ and } \boldsymbol{Q} = 40 \text{ units}$$
$$= 32{,}000 - [1{,}000 + 5(1{,}600)]$$
$$= 32{,}000 - [1{,}000 + 8{,}000]$$
$$= 32{,}000 - 9{,}000$$
$$= \$23{,}000$$

Alternatively, you can solve for profit without first solving for price $\boldsymbol{P}$ by substituting the demand equation for $\boldsymbol{P}$ into the total revenue expression.

$$\pi = \text{TR} - \text{TC}$$
$$= (\boldsymbol{Q})(\boldsymbol{P}) - [1{,}000 + 5(\boldsymbol{Q})(\boldsymbol{Q})]$$
$$= (\boldsymbol{Q})(1{,}200 - 10\boldsymbol{Q}) - [1{,}000 + 5(\boldsymbol{Q})(\boldsymbol{Q})] \qquad \text{after substituting the demand equation } \boldsymbol{P} = 1{,}200 - 10\boldsymbol{Q} \text{ for } \boldsymbol{P}$$
$$= (40 \text{ units})[1{,}200 - 10(40 \text{ units})] - [1{,}000 + 5(40 \text{ units})(40 \text{ units})] \qquad \text{after plugging in } \boldsymbol{Q} = 40 \text{ units}$$
$$= (40)(1{,}200 - 400) - [1{,}000 + 5(1{,}600)]$$
$$= (40)(800) - [1{,}000 + 8{,}000]$$
$$= 32{,}000 - 9{,}000$$
$$= \$23{,}000$$

4. Solving for the level of output $\boldsymbol{Q}$ (perfectly competitive) or the output of a socially efficient firm that is producing at the level equivalent to that of a firm in a perfectly competitive market.

$$MC = \boldsymbol{P}$$
$$10\,\boldsymbol{Q} = 1{,}200 - 10\boldsymbol{Q} \qquad \text{after substituting the demand equation } \boldsymbol{P} = 1{,}200 - 10\boldsymbol{Q} \text{ for } \boldsymbol{P}$$
$$10\,\boldsymbol{Q} + 10\,\boldsymbol{Q} = 1{,}200 \qquad \text{after adding } 10\boldsymbol{Q} \text{ to both sides of the equal sign}$$
$$20\,\boldsymbol{Q} = 1{,}200$$
$$\boldsymbol{Q} = 1{,}200/20$$
$$\boldsymbol{Q} = 60 \text{ units}$$

Note: The perfectly competitive firm's output is higher than that of the monopolist's.

5. Solving for the price $\boldsymbol{P}$ charged by the perfectly competitive firm (socially efficient firm) while producing at its profit-maximizing level of output $\boldsymbol{Q}$.

$$\boldsymbol{P} = 1{,}200 - 10\boldsymbol{Q}$$
$$= 1{,}200 - 10(60 \text{ units})$$
$$= 1{,}200 - 600$$
$$\boldsymbol{P} = \$600$$

6. One way of solving for the competitive firm's (or the socially efficient firm's) profit, π.

π = TR – TC
= (P)(Q) – [1,000 + 5(Q)(Q)]
= ($600)(60 units) – [1,000 + 5(60 units)(60 units)] after plugging in P = $600 and Q = 60 units
= 36,000 – [1,000 + 5(3,600)]
= 36,000 – [1,000 + 18,000]
= 36,000 – 19,000
= $17,000

Alternatively, as in Number 3 above, you can solve for profit without first solving for price P by substituting the demand equation for P into the total revenue expression.

π = TR – TC
= (Q)(P) – [1,000 + 5(Q)(Q)]
= (Q)(1,200 – 10Q) – [1,000 + 5(Q)(Q)] after substituting the demand equation P = 1,200 – 10Q for P
= (60 units)[1,200 – 10(60 units)] – [1,000 + 5(60 units)(60 units)] after plugging in Q = 60 units
= (60)(1,200 – 600) – [1,000 + 5(3,600)]
= (60)(600) – [1,000 + 18,000]
= 36,000 – 19,000
= $17,000

7. At this point, it's a good idea to label the graph in Fig. 1 by inserting into those empty, tiny circles the numbers for prices and quantities each for monopoly and perfect competition. This makes it easier to follow the solutions below.

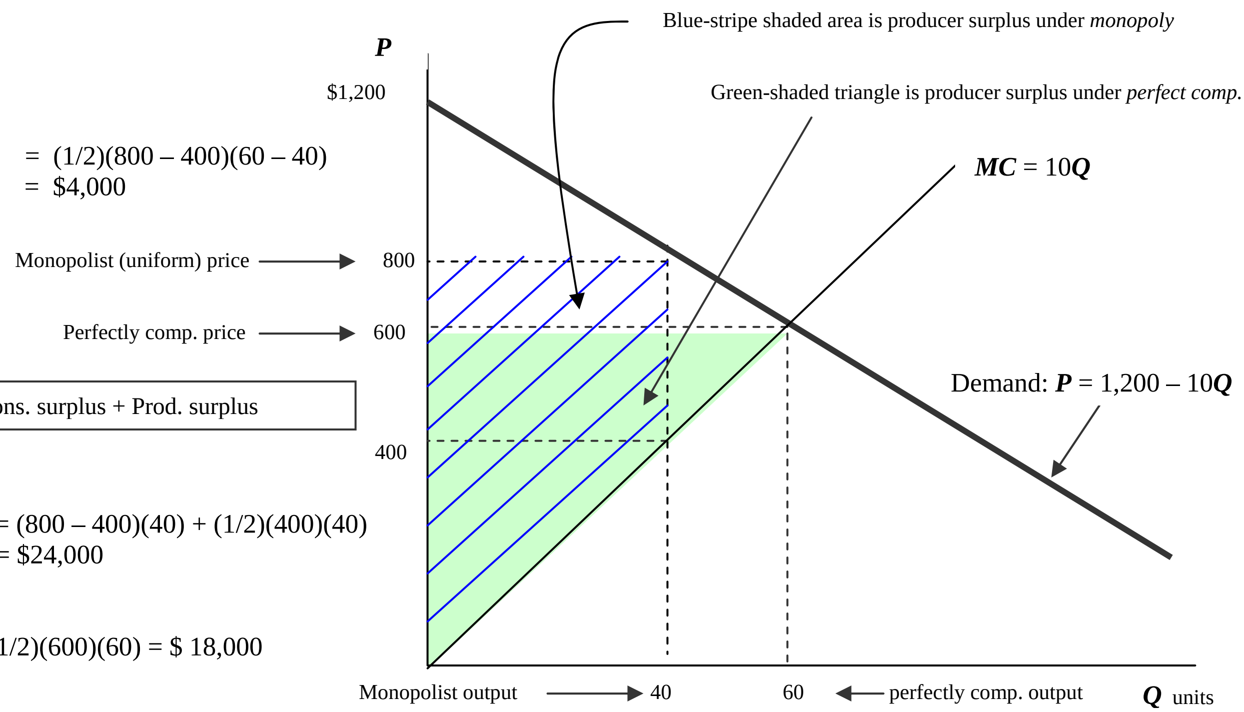
Note: Area of a triangle = (1/2)(base)(height)

Solving for *consumer surplus* under *monopoly*: (It is the yellow-shaded triangular area in the graph.)	Solving for *consumer surplus* under *perfect competition*: (It is the stripe-shaded triangular area in the graph.)
Cons. surplus under monopoly = (1/2)($1200 – $800)(40 units) = $8,000	Cons. Surplus under perfect. comp. = (1/2)($1200 – $600)(60 units) = $18,000

Note: The *loss* in *consumer surplus* as a result of monopoly (with uniform pricing) as compared to perfect competition is = $18,000 – $8,000 = $10,000.

Fig. 2

Deadweight loss under *monopoly* = (1/2)(800 – 400)(60 – 40)
= $4,000



Total surplus = Cons. surplus + Prod. surplus

Prod. surplus under monopoly = (800 – 400)(40) + (1/2)(400)(40)
= $24,000

Prod. surplus perfect comp. = (1/2)(600)(60) = $ 18,000

Alternatively, deadwt. loss = tot. surplus under perfect comp. – tot. surplus under monopoly

Detailed Solutions (Part 2)

Monopoly With Price Discrimination

Note: Under monopoly with perfect price discrimination, the monopolist is assumed to know the price that each of its customer is willing to pay (called *reservation price*) and so charges each of them accordingly, i.e., each customer is charged a different price. This is shown by the blue stripes in Fig. 3 below. So for the price-discriminating monopolist, the total revenue area is represented by the blue-stripe shaded area.

Moreover, the output of the price-discriminating monopolist is equal to the output of the perfectly competitive firm, which is 60 units in this case, because the monopolist is now able to capture *all* of the consumer surplus previously due to the consumers under perfect competition.

1. Calculating *Total Revenue* (under monopoly with perfect price discrimination):

TR = triangle representing consumer surplus under perfect competition + rectangular area bounded by $600 and 60 units
= (1/2)(1200 – 600)(60 units) + ($600)(60 units)
= $18,000 + $36,000
= $54,000

TC = $1{,}000 + 5\boldsymbol{Q}^2$
= $1{,}000 + 5(60 \text{ units})^2$
= $19,000

Consumer surplus under monopoly with perfect *price discrimination* = 0

profit, π = TR – TC
= $54,000 – $19,000
= $35,000

2. Calculating *Producer Surplus* (under monopoly with perfect price discrimination):

Producer Surplus = green-shaded triangular area
= (1/2)(1,200 – 0)(60) = $36,000

Alternatively, Producer Surplus = Total Revenue – triangular area are bounded by $600 and 60 units
= $54,000 – (1/2)(600)(60) = $36,000

Fig. 3

Deadweight loss under *monopoly* with perfect *price discrimination* = 0

Total surplus = Cons. surplus + Prod. surplus

Prod. surplus under *monopoly* with perfect *price discrimination* = (1/2)(1,200 – 0)(60) = $36,000